NVH Calculations for Drivetrains, A Methodology for Selection of the Best Suitable Calculation Approach for a Specific Purpose

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Abstract

In this paper, some guidelines are given how to apply the various calculation approaches for different purposes in the field of dynamic evaluation. For this purpose, a two-stage powertrain model powering the front wheels of an electric vehicle is considered as a typical gearbox. Two different variants of gear designs regarding the microgeometry are considered for this study. These two variants consist of the “best-of-best" (BOB) design, where the modifications lead to the minimum peak-to-peak transmission error (PPTE), and secondly the “worst-of-worst" (WOW) design, which represents the worst cumulation of the specified tolerances as per manufacturing drawings. Then, by conducting the forced response analysis in the KISSdesign module of KISSsoft, the bearing reaction forces of both models in the time and frequency domains are calculated. For characterization of the emitted noise of the WOW and BOB models, the bearing forces of the forced response calculated in KISSsoft are imported to RecurDyn and applied to the housing of the powertrain model. In this way, one can explicitly investigate the effect of the microgeometry modifications and the improvement of the BOB model compared to the WOW model in the level of noise emitted directly from the housing surface. The results confirm that the microgeometry modifications lead to the lower level of the noise emission by reducing the peak-to-peak transmission error and subsequently lower excitations to the powertrain system.

Introduction

The performance evaluation of powertrains and their vibration characterization under dynamic loadings are becoming more and more state of the art in the engineering of drivetrains. In many cases, engineers and designers require a rating of the noise for the overall transmission to clearly specify the sound pressure level emitted from the housing surface. However, in some cases, only the evaluation of the overload forces at meshing gears through a forced response analysis is sufficient for the evaluation of the gears quality and performance.

A major advantage of the dynamic calculation is to check the drivetrains for critical frequencies which can create both noise and overloaded excitations, whereas the static calculation during the dimensioning and sizing process does ignore the influences of the dynamics such as inertias and frequencies. The basic principles of the dynamic calculation are the same for both the noise and overloading evaluation. Based on the forced excitation imposed from gears, shafts, torque ripple, etc., the forced response analysis is performed and at the bearing positions, the transient loads are evaluated. In case of the NVH evaluation, furthermore the housing is excited by the transient bearing forces, and consequently, the noise emitted from the housing is evaluated.

The noise emission level of a gear set is directly related to the excitation behavior of the powertrain. The Transmission Error (TE) is known as the key factor affecting the excitation behavior. The dynamic interaction between tooth mesh and the structural behavior of the drive train characterizes the noise and vibration behavior. Furthermore, the deflection of the transmission housing arising from the operational loads can result in misalignment of the shafts, bearings, and gears. This fact in turns creates audible noises by affecting the transmission error [1]. With increasing the input speed of the system, the excitation frequencies of the gear sets are increased. The dynamic loads are significantly enhanced when the gear mesh frequencies coincide with any of the system eigenfrequencies [2]. Therefore, analysis of the high-frequency NVH responses require the inclusion of component flexibility into the impulsive transient analysis [3, 4]. An effective modeling approach is proposed in [5] to approximately predict the dynamic bearing loads and the housing surface acceleration based on the results from lumped parameter gear dynamic simulations. In this context, since any particular tooth modification can be valid for a certain operating load range, the study presented in [6] analyzes the forced responses for several applied mean torque load cases. The required modifications applied on mating gears are interacting, so the decision of which modification to add or to change is a difficult task. To remedy this drawback, a strategy to find the optimum combination of modifications with a fast, straightforward procedure is developed in [7].

In KISSdesign module of KISSsoft, a toolkit for analysis of drivetrains by performing the forced response analysis is implemented. Based on the static transmission error of the gears, shaft imbalances, etc., the transient bearing loads are calculated considering the inertias and masses. As the approach is based on analytical methods, the calculation is very fast. This allows the engineer to evaluate the gears in shortest time not only with respect to the static parameters, but also the dynamic effects for any of the excitation frequencies. Simultaneously, the manufacturing errors of gears can be evaluated and their effect on the dynamic properties, by means of using measured gear flank data, can be further investigated.

For characterization of the NVH properties, a calculation process using KISSdesign and RecurDyn software has been developed. When reading the transient bearing forces from KISSdesign into RecurDyn and applying them to the housing, an overall rating of the transmission regarding NVH can be carried out.

The design engineer needs to find the most efficient process to rate the dynamic behavior of his transmission. If the focus is on the overloading effects on gears and the quick estimation of emitted noise, the KISSdesign tool is very efficient. For an overall and final rating of the drivetrain and housing, the evaluation in RecurDyn gives a complete result. Both methods can be used in the case of EV drives and the effect of various microgeometry gear modifications on both, the gear dynamic overloading and the noise emission of the housing is further investigated.

A two-stage powertrain model

The electric axle analyzed in the following is a single-speed, two-stage gearbox powering the front wheels of an electric vehicle, as shown in Figure 1. The powertrain consists of the input shaft, intermediate shaft and output shaft. For the sake of briefness, the depicted numbering shown in Fig. 1 for the bearings and gears are used from now on within the manuscript.

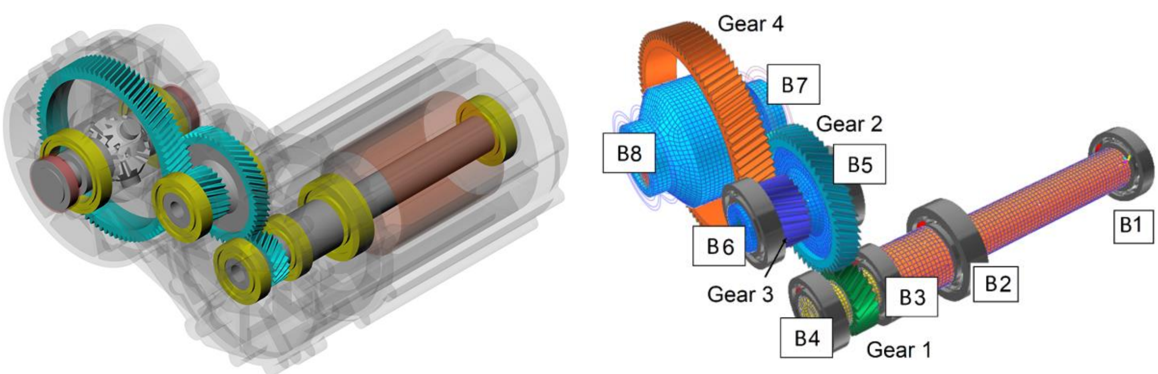

Fig. 1: Two-stage gearbox model layout (left) and bearings and gears numbering (right)

The output gear stage is integral to the differential case. The modeling of the differential stage is not considered in the present paper. Some details of the model can also be found in [8].

Model setup in KISSsoft

The damping model of KISSsoft at bearings is based on the inclusion of separate viscous damping in translational and rotational directions. In this mode, the same damping coefficients are used for all bearings. For the translational directions, C_{ux} = C_{uy}= C_{uz}=100 Ns/m, and for the rotational directions, C_{rx}=C_{ry}=C_{rz}=1 Nms/rad. Higher amount of damping will considerably damp the vibrations and will lead to unrealistic response of the system. Therefore, selection of damping coefficients has to be carried out with care.

Table 1 reports the gears macrogeometry data for the two designs (BOB and WOW models) with a transverse contact ratio ε_α>2. For both designs, first stage gear mesh order is 23.00 and the second stage gear mesh order is 9.98. Order 1 is referred to electric machine shaft.

Table 1: Gears data and meshing orders

Gear parameters		I stage		II stage	
		Gear 1	Gear 2	Gear 3	Gear 4
Number of teeth	z [-]	23	53	23	89
Helix angle at reference circle	β [°]	30		15	
Normal pressure angle	α_n [°]	20		20	
Normal module	m_n [mm]	2.5		2.6	
Profile shift coefficient	x^* [-]	0.0163	-0.6682	0.4706	-0.6659
Face width	b [mm]	25	23	40	38
Center distance	a [mm]	107.99		150.22	
Gear Mesh Excitation order	GMF	23.00		9.98	
Gear profile HCR	$h^*_{fP}/\rho^*_{fP}/h^*_{aP}$	1.80/0.19/1.35	1.60/0.29/1.60	1.60/0.29/1.35	1.60/0.29/1.45
Transverse contact ratio HCR	ε_α	2.05		2.10	
Overlap ratio	ε_β	1.46		1.20	

Microgeometry variants

The simulations are carried out for performance improvement evaluation of the HCR gears regarding the peak-to-peak transmission error (PPTE), gear meshing and bearing forces. Two microgeometry modifications (Tables 2 and 3) are designed with the help of a system-level tool: helix angle modification and crowning are adopted to reduce the face load factor KHβ under various load conditions, while tip relief and profile crowning are adopted to eliminate contact shock and reduce the PPTE. The two variants consist of the “best-of-best" (BOB) design, where the modifications lead to minimum PPTE, and secondly the “worst-of-worst" (WOW) design, which represents the worst cumulation of the specified tolerances as per manufacturing drawing. Details on the methodology to design gear microgeometry are reported in [5].

Table 2: Microgeometry modifications for HCR gearset, Best-Of-Best (BOB)

Microgeometry modifications [μm] - BOB	I stage		II stage	
	Gear 1	Gear 2	Gear 3	Gear 4
Flank crowning C_β	8	8	6	6
Tip relief, arc like (Long) $C_{\alpha a}$	10 (d_{Ca} = 69.20 mm)	10 (d_{Ca} = 151.43 mm)	4 (d_{Ca} = 66.39 mm)	4 (d_{Ca} = 241.43 mm)
Profile crowning C_α	5	5	8	8

Table 3: Microgeometry modifications for HCR gearset, Worst-Of-Worst (WOW)

Microgeometry modifications [μm] - WOW	I stage		II stage	
	Gear 1	Gear 2	Gear 3	Gear 4
Flank crowning C_β	11	11	9	9
Tip relief, arc like (Long) $C_{\alpha a}$	13 (d_{Ca} = 69.20 mm)	13 (d_{Ca} = 152.43 mm)	7 (d_{Ca} = 66.39 mm)	7 (d_{Ca} = 241.43 mm)
Profile crowning C_α	8	8	11	11

Table 4 reports the contact analysis results of both BOB and WOW models. The analysis is carried out for the constant input torque of 320 Nm applied to the input shaft. As it can clearly be seen, the peak-to-peak transmission error and meshing excitation force for both gear pairs in the BOB model are decreased when compared to the WOW model.

Table 4: Contact analysis results of both the BOB and WOW models

Parameter	Gear pair	Model type	Min. value	Max. value	Peak-to-peak	Mean value
Transmission Error (μm)	Gear1- Gear2	BOB	-33.5448	-32.7279	0.8169	-33.1788
		WOW	-37.3705	-36.1790	1.1915	-36.7387
	Gear3- Gear4	BOB	-37.3351	-36.7231	0.6121	-36.9384
		WOW	-40.6329	-39.8120	0.8209	-40.2578
Excitation force (N)	Gear1- Gear2	BOB	10287.8498	10661.4127	373.5625	10453.7774
		WOW	10177.1898	10701.8429	524.6531	10455.7443
	Gear3- Gear4	BOB	25105.1113	25648.7881	543.6767	25455.7978
		WOW	25124.0984	25844.9680	720.8696	25455.1095
Tangent stiffness (N/μm)	Gear1- Gear2	BOB	417.5965	459.6642	42.0677	438.1450
		WOW	382.6402	424.3360	41.6958	407.3021
	Gear3- Gear4	BOB	815.5913	865.8156	50.2243	840.9703
		WOW	798.8536	841.4226	42.5690	825.5980
Secant stiffness (N/μm)	Gear1- Gear2	BOB	311.7353	319.4610	7.7257	315.1696
		WOW	279.7836	289.0391	9.2554	284.6800
	Gear3- Gear4	BOB	681.7263	693.1833	11.4570	689.1499
		WOW	626.4299	639.4440	13.0141	632.4079

However, the mean value of the excitation force does not imply any significant difference in both models. This is especially due to the fact that the mean value of the meshing stiffness of both gear pairs in the BOB model has higher values than that of the WOW model. In addition, the tangent stiffness is the local stiffness at operating point whereas the secant stiffness represents the overall stiffness during meshing.

Figure 2 illustrates the excitation force for both gear pairs. It can be observed that the excitation force in the BOB model has a superior trend regarding undesired vibrations and NVH analysis characteristics. This important feature will be shown later when conducting the NVH analysis for both the BOB and WOW models.

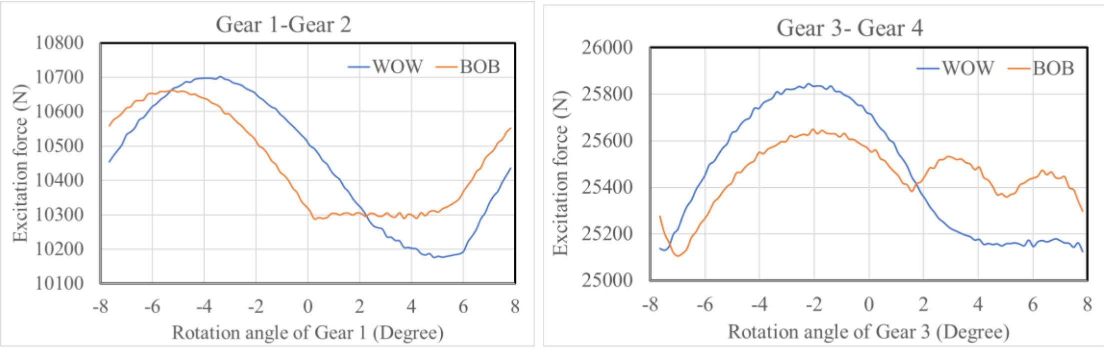

Fig. 2: Excitation force of meshing gear pairs in the BOB and WOW models

Forced response analysis in KISSsoft

The powerful and user-friendly user interface of the forced response analysis in KISSsoft allows the analysists and engineers to quickly and efficiently perform the dynamic analysis of the powertrain systems. Within this tool, a comprehensive list of different settings and options are provided which enable the user to precisely investigate the vibration characteristics of the system. With the current implementation, the forced response analysis of powertrains with helical and bevel gears mounted on single and coaxial shafts with switchable or coupling connections elements can be accomplished. The theory of the forced response analysis in KISSsoft is based on the research work done by Beermann [10] and others. The basic approach is, that the excitation forces of the meshing gears are calculated and then are applied to the system according to their excitation orders. The total procedure is based on the frequency response analysis where all excitations and responses are represented in terms of the excitation frequencies together with their corresponding amplitudes and phase angles.

Three types of the excitations including the unbalanced mass forces, gear mesh forces resulting from peak-to-peak transmission error and variable nonlinear meshing stiffness, and externally applied torque ripples are available. For a better imagination of the powertrain system responses, the 3D data are generated for visualization. Based on this feature and by setting a proper scale factor, movement of different parts of the system can be seen at all running input shaft speeds (or more specifically at all excitation frequencies). Starting from the first harmonic of excitation, one can include a desired number of excitation harmonics. This is particularly of great importance to select a suitable number of harmonics to make a trade-off between a demanded level of the results accuracy and the analysis effort. As another interesting issue, two different dynamic modeling approaches can be selected. When modeling the flexible shafts, one may consider only the torsional responses or include both the bending and torsional responses.

The main output of the forced response analysis is the variation of the dynamic factor for a given range of input shaft speeds. The dynamic factor, which is typically defined as the ratio of the maximum dynamic excitation loading between the meshing gears to the static contact force, characterizes the system behavior under dynamic loading at different shaft speeds and reveals the margins of the operational speeds for which the powertrain system can be significantly excited. In addition to the dynamic factor, some other important outputs in graphical representations as well as in the form of text output files are generated to allow the user to evaluate and interpret the results quickly and easily. These results, which include shaft deformations and forces at any desired cross section, meshing gears contact outputs, bearing reaction forces in time and frequency domains, can be further integrated to other commercial and research software tools for NVH analysis.

Input parameters setup in the forced response analysis

In this analysis, the input shaft rotational speed is 1000 rpm, and a constant torque of 320 Nm is applied to the input coupling. This speed is selected to assure that the excitation frequencies are in subcritical areas which allows more precise comparison between two calculation approaches. This is not a recommended speed for an NVH analysis of the transmission. The material damping of the flexible shafts in the torsional, axial and bending directions is $10^{(-5)}$ s. Furthermore, the gear mesh damping is 2500 Ns/m.

In order to select a proper harmonics number, starting from the first harmonic of excitation, the forced response analysis is carried out for the speed range from 100 to 12000 rpm. Subsequently, the number of excitation harmonics is gradually increased, and the analysis is repeated. The results of this investigation are shown in Figs. 3 and 4 for the WOW and BOB models, respectively. According to the results, one can easily conclude that for the majority of the speed range, the dynamic factors obtained with Nh=2 and Nh=3 coincide each other. Further increase of the harmonics number will not considerably change the dynamic factor.

This implies that the results based on Nh=2 will lead to an acceptable trade-off between a demanded level of the accuracy and the analysis effort at running speed of 1000 rpm.

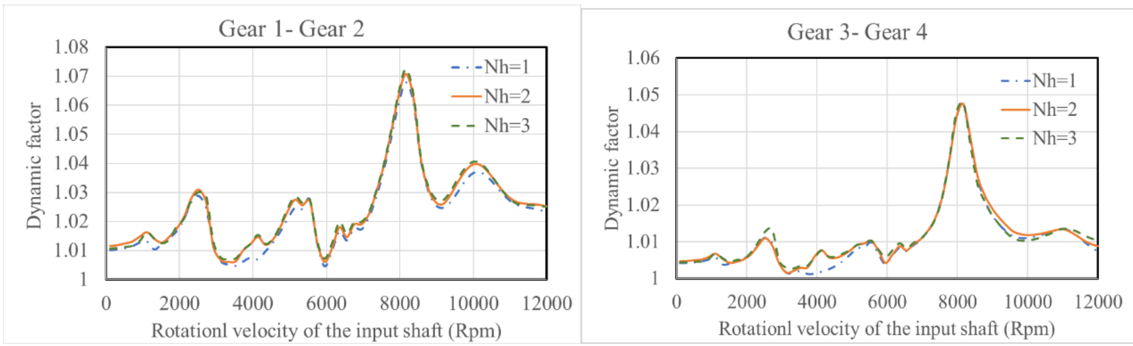

Fig. 3: Dynamic factor of meshing gear pairs in the WOW model

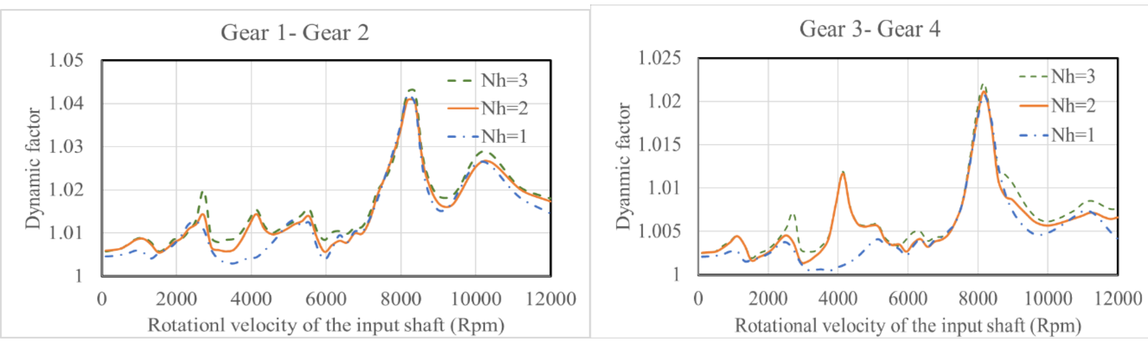

Fig. 4: Dynamic factor of meshing gear pairs in the BOB model

Results and discussion

The forced response analysis is carried out in the KISSdesign module of KISSsoft and the bearing reaction forces in time and frequency domains are calculated. Figures 5 and 6 illustrate the force and moment magnitudes for bearings 6 and 7, respectively. The results demonstrate the improvement of the reaction forces in the BOB model with respect to the WOW model. The reduction of the reaction forces and moments is directly related to the reduction of the meshing excitation forces in the BOB model due to the peak-to-peak transmission errors modification.

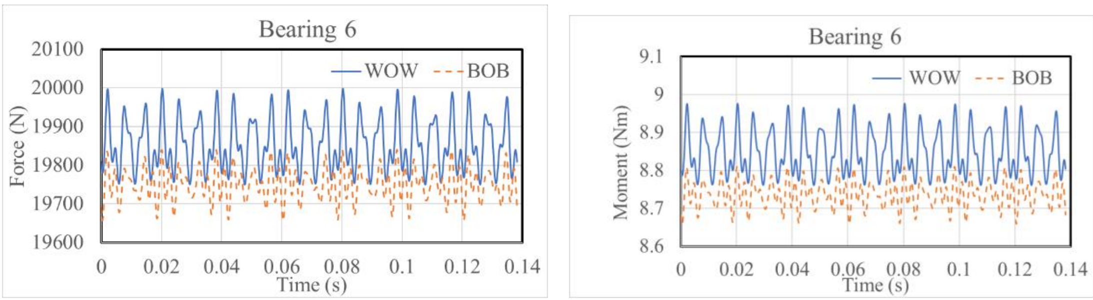

Fig. 5: Force and moment of bearings 6 in time domain calculated in KISSsoft

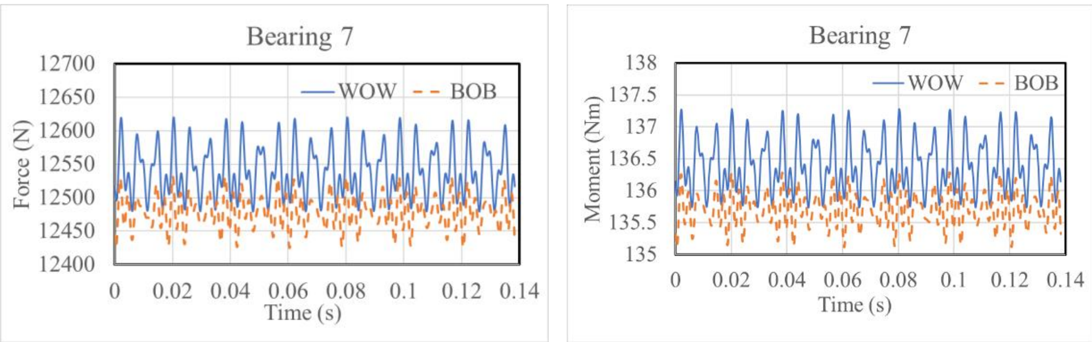

Fig. 6: Force and moment of bearings 7 in time domain calculated in KISSsoft

The force amplitudes of bearings 6 and 7 in frequency domain are shown in Fig. 7. The excitation orders 23 and 46 from the first gear pair and 53 and 106 from the second gear contribute clearly to the force amplitudes in the WOW and BOB models. Since in this study only the first two harmonics with Nh=2 is considered, the amplitudes corresponding to these orders are apparent. The meshing frequencies of the first and the second gear pair for the input rotational velocity of 1000 rpm are $f_{Gear1\text{-}Gear2}$= 2408.55 rad/s=383.33 Hz and $f_{Gear3\text{-}Gear4}$= 1045.22 rad/s=166.35 Hz, respectively. Therefore, the excitation frequencies associated with the first harmonic arising from the first and the second gear pair are 383.33 Hz and 166.35 Hz, respectively.

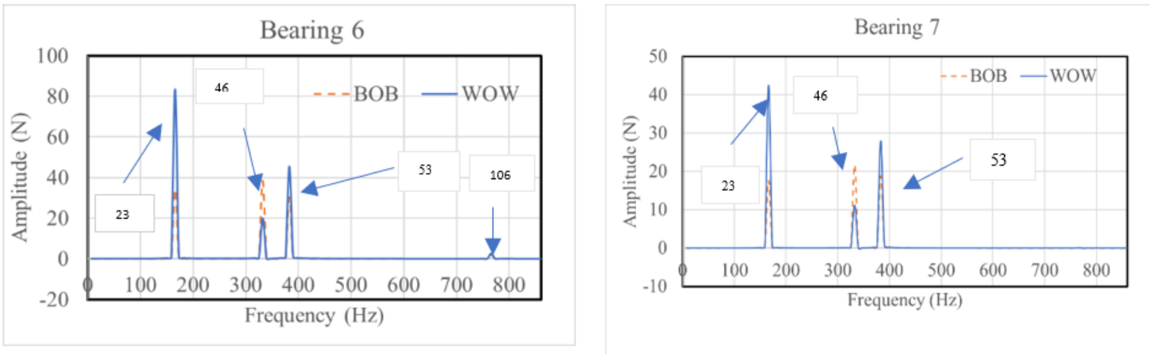

Fig. 7: Force amplitudes of bearings 6 and 7 in the WOW and BOB models

Table 5 reports the results of this study for the first two harmonics in more details.

Table 5: Force amplitudes of bearings 6 and 7 in KISSsoft

Model type	Gear pair	Order No.	Frequency (Hz)	Frequency amplitude	
				Bearing 6	Bearing 7
WOW	Static force	0	0	19854.16	12538.99
	Gear3-Gear4	23	166.35	83.42	42.46
		46	332.70	20.03	11.18
	Gear1-Gear2	53	383.33	45.46	27.95
		106	766.66	2.66	0.148
BOB	Static force	0	0	19750.83	12478.78
	Gear3-Gear4	23	166.29	34.33	17.68
		46	332.70	38.35	21.71
	Gear1-Gear2	53	383.19	30.33	18.89
		106	766.66	2.13	0.0739

Forced response analysis in RecurDyn

In this section, firstly the parameters specification of the model in RecurDyn is introduced. Then, for both the WOW and BOB models, the same geometry of the shafts, gear bodies and bearings are used in the dynamic analysis with the same inputs as those which have been used in KISSsoft.

It is noticeable to mention that the damping model of the bearings in RecurDyn is different than the model in KISSsoft. RecurDyn automatically calculates the damping forces using the damping ratio ζ, the bearing stiffness matrix K, and the damping exponent n. In this approach, the damping matrix is defined with $\mathbf{C} = \zeta \mathbf{K}^n$. Here, for all bearings we set $\zeta=0.001$, and $n=1$. Another important difference is that the forced response analysis in KISSsoft is carried out in the frequency domain and then the results are transferred to the time domain. On the other hand, in RecurDyn, all simulations are done in time domain. In addition, the flexibility of the shafts in RecurDyn is based on the modal analysis of the finite element model of the shafts. However, in KISSsoft the analytical model of the flexible shafts is based on the theory of Timoshenko and Euler beams.

Figures 8 and 9 illustrate the force and moment magnitudes for bearings 6 and 7, respectively calculated in RecurDyn. The results confirm that the mean value of the reaction force and moment in the BOB model is reduced with respect to the WOW model. However, apparently higher fluctuations of the reactions force and moment in the BOB model can be observed.

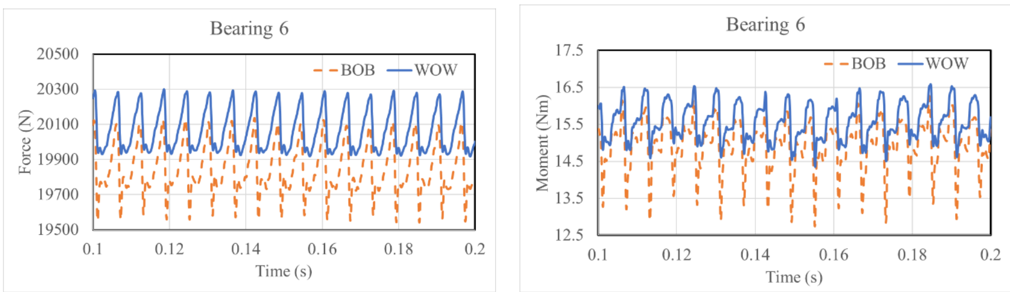

Fig. 8: Force and moment of bearings 6 in time domain calculated in RecurDyn

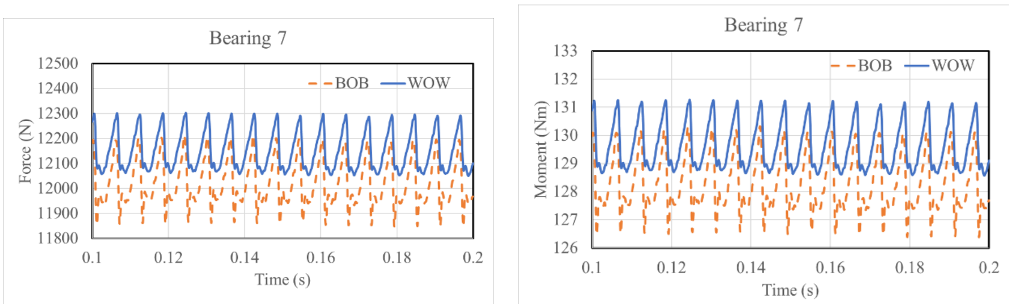

Fig. 9: Force and moment of bearings 7 in time domain calculated in RecurDyn

For a better comparison of the results of the forced response analysis in RecurDyn and KISSsoft, Table 6 reports the force amplitudes of bearings 6 and 7 in frequency domain for different excitation harmonic orders in more details. In general, the results are in good agreement with KISSsoft. However, due to the differences explained in the simulation methodology and the parameters setup, some deviations can be observed.

Table 6: Force amplitudes of bearings 6 and 7 in RecurDyn

Model type	Gear pair	Order No.	Frequency (Hz)	Frequency amplitude	
				Bearing 6	Bearing 7
WOW	Static force	0	0	20034.39	12124.07
	Gear3-Gear4	23	166.35	75.97	50.32
		46	332.70	23.16	14.52
		69	499.05	19.15	12.03
	Gear1-Gear2	53	383.33	3.80	1.48
		106	766.66	2.46	1.22
BOB	Static force	0	0	19796.35	11984.35
	Gear3-Gear4	23	166.35	76.84	52.15
		46	332.70	41.36	25.14
		69	499.05	38.10	23.88
	Gear1-Gear2	53	383.33	6.74	1.01
		106	766.66	3.12	1.00

NVH analysis in RecurDyn based on the forced response analysis in KISSsoft

In previous sections, the forced response analysis of both BOB and WOW models in KISSsoft is explained. As the result of this analysis, the bearing forces in time domain are calculated. To validate the results of our approach, the dynamic analysis of the same models in RecurDyn was performed.

In this section, the bearing forces of the forced response are imported to RecurDyn and applied to the housing of the powertrain model. The shafts and gears are not included in the model. Therefore, one can explicitly investigate the effect of the microgeometry modifications and the improvement of the BOB model compared to the WOW model in the level of noise emitted directly from the housing surface.

The analysis starts with setting the parameters of the housing and extracting a reduced order flexible model in a modal analysis approach by selecting the first 200 eigenfrequencies. Then, the bearing forces are applied as time dependent spline functions at the bearing position of the housing. Before to conduct any NVH analysis, the housing responses are calculated in a dynamic analysis. As a major kinematic parameter required for the NVH analysis, the surface velocities at nodes of the meshed geometry are calculated. Finally, the equivalent radiated power (ERP), as the main factor for measuring the emitted noise level from the housing surface to the environment is calculated. The ERP is defined as follows:

$$e_{ERP} = f_{RLF} \cdot \frac{1}{2} \cdot C \cdot \rho \cdot \sum(A_i \cdot v_i^2)$$

where f_{RLF} is the radiation loss factor, C is the sound velocity, ρ is the density of a target material which transfers the noise, e.g. air, A_i is the area on the i-th flexible panel of the meshed surface, and v_i is its face normal velocity. Further details can be found in [11]. In order to clearly demonstrate which parts of the housing surface emits higher level of the noise, the contour plot of the ERP is very helpful. It can subsequently be used to address demanded design modifications, such as local stiffening of the housing by means of the ribs, to reduce the vibrations. Figure 10 shows ERP contours of the WOW and BOB models at t=0.118 (s). It is clearly seen that in both models, the region of the housing close to the output shaft has higher ERP values, and consequently, emits considerable noise to the environment. In addition, the BOB model has a superior performance with respect the noise emission. However, it is noticeable to mention that the variation of the ERP depends on the simulation time and on the local evaluated position on the housing. This may lead to cases when the WOW model may have locally equal or less noise emission than the BOB model. The differences of the results obtained here are due to the very small differences in the flank modifications of the gear designs (tolerances of ±3 μm), see Tables 2 and 3. Also an analysis within the microgeometry tolerances depends significantly on the operation conditions under which the system is running.

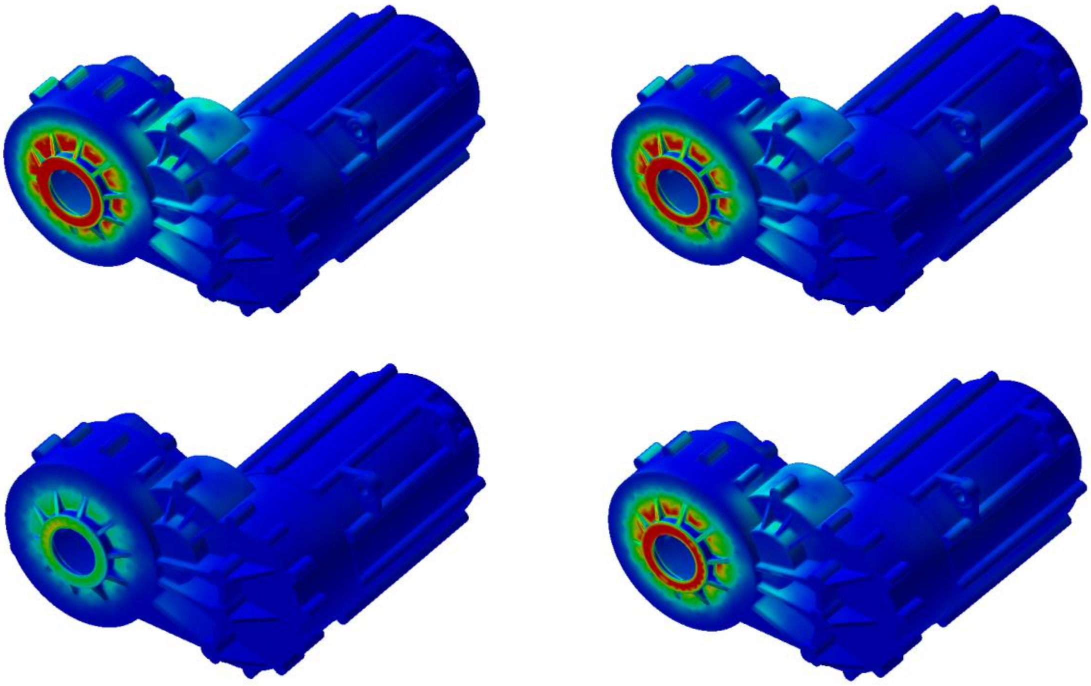

Fig. 10: ERP contours of the WOW (top) and BOB models (bottom) at t=0.118 (s) (left) and t=0.14 (s) (right)

To further clarify the performance of both models, the ERP in time and frequency domains are plotted in Fig. 11. According to the results, the higher contribution of the WOW model with respect to the BOB model in noise generation can be observed. These results confirm that the microgeometry modifications lead to the lower level of the noise emission by reducing the peak-to-peak transmission error and subsequently lower excitations to the powertrain system.

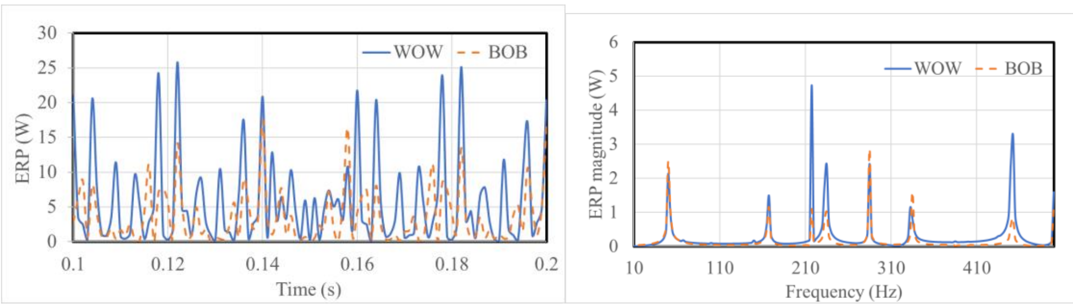

Fig. 11: ERP of the WOW and BOB models in time and frequency domains

Conclusion

The calculation process for noise emitted from the housing is an established process, where the transient bearing forces are determined from the various vibration sources within the powertrain system such as gears, torque ripple and others. These transient bearings are the source for the excitation of the housing for vibration and hence the noise emission. The calculation process itself can be done through MBS software in time domain, but it is also possible within dedicated drivetrain design software using the frequency domain approach.

One major part within the overall NVH calculation is the calculation of gear contact forces. The variation of these forces is most relevant for the transient bearing loads. Therefore, it is important to consider the gear macro and microgeometry properly.

In this paper, it was shown, that it is possible to calculate the housing excitation of drivetrains reliably by the approach of frequency domain, also considering microgeometry of gears, such as tip relief, crowning etc. Also, it was shown that the designs between optimal designs ("best-of-best, BOB") compared to the gear topologies including the worst combination of tolerances ("worst-of-worst", WOW), show significant lower emitted radiated power of the housing.

The calculation based on analytical approaches may have some lack in detail definition of e.g. modal damping. However, the calculation of the gear contact forces is typically much more precise by the gear contact analysis tool. This allows the engineers to use the same software tool for design and strength rating, also for the calculation of transient bearing forces. It is also possible to connect the transient bearing forces to the MBS tool, where the loads are applied to the housing directly. This also allows to have a qualitative rating of the housing noise emission, and to find weaknesses of the housing regarding local vibrations. Such undesired vibrations are disturbing during the operation of the transmission, and which may be improved in the design.

As an outlookof this approach, KISSsoft also plans to establish a 'design-manufacture-measure' loop, where the manufactured flank topologies are applied in the calculation of the gear contact forces. This will allow to do an assessment of manufacturing errors in terms of the noise emission and to avoid having disturbing gears within the transmission during the simulation, and before any detection in the final acceptance tests and a costly disassembling of transmission.

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